Quantum processes in the field of a two-frequency circularly polarized plane electromagnetic wave

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We solve Dirac's equation for an electron in the field of a two-frequency plane electromagnetic wave, deriving general formulas for the probabilities of radiation of an electromagnetic wave by the electron, and for the probabilities for pair production by a photon when the two-frequency wave is circularly polarized. In contrast to the case of a monochromatic-plane electromagnetic wave, when an electron is in the field of a two-frequency circularly polarized wave, the emission rates of various ''biharmonic photons'' are affected by interference between the two waves. When a high-energy photon is in such a field, similar interference effects arise in the process of pair production. [S1063-651X(98)04802-8]

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I. INTRODUCTION

There have been many investigations of multiphoton processes in strong electromagnetic fields since the invention of the laser in the early 1960s. Reiss formulated the ''(multi)photon'' absorption processes that generate electron pairs when a photon collides with an intense wave field [1]. Nikishov and Ritus derived formulas both for the ''(multi-)photon'' absorption of an electron in a plane electromagnetic field, and the ''(multi)photon'' absorption processes of pair production [2]. Narozhny *et al.*. extended these formulas to the case of a circularly polarized electromagnetic wave [3]; such effects were observed recently [4]. The reason we enclose ''(multi)photon'' in quotation marks is discussed in Sec. III. Other theoretical studies made by the classical approach [5,6] and by the semiclassical scattering theory as reviewed by Ehlotzky [7], or by quantum electrodynamics (QED) [8], all of which describe the ''(multi)photon'' processes of harmonic-photon generation, have contributed enormously to advances in our understanding. The predicted second harmonic-photon emission phenomenon was observed by Englert and Rinehart [9]. Besides these simple harmonics of a monochromatic wave, it is possible to generate various ''biharmonics'' by using a two-frequency intense laser beam. We note that Puntajer and Leubner considered the specific case of this problem by classical electrodynamics [10] (constraining the frequency of the two waves $\omega_2 = 2\omega_1$; the wave and electron beam are counterpropagating). In the present paper, we consider a more general case within the QED theory, with an arbitrary frequency for the two waves and an arbitrary angle between the wave and electron beam.

The theoretical research with QED on ''(multi)photon'' absorption processes in an intense laser beam is based on Dirac's equation. The success of the QED theory in describing such processes demonstrates that the semiclassical treatment of the laser beam as an external unquantized electromagnetic field is a good approximation. Using the semiclassical treatment of electromagnetic fields for an electron in a plane electromagnetic wave, an exact solution of Dirac's equation was found [11]. However, an exact solution of this equation for more than one plane wave propagating in arbitrarily different directions has not been obtained, and it seems difficult to do so because generally $(k_1 k_2) \neq 0$ (k_1 and k_2 are four-vector momenta of two plane waves) so that the Dirac equation cannot be solved by a simple integral. To remove this complication but, at the same time, not lose generality in physics, we consider a two-frequency plane wave; that is, two different plane waves propagating in the same direction. With this approach, we can find an exact solution of Dirac's equation for an electron in these waves.

In this paper, we describe our study of a two-frequency plane electromagnetic wave with four-vector momenta, k_1 and k_2, which interact with a single free electron. This approach led us to a new prediction of ''(multi)photon'' absorption and emission processes, and of ''biharmonic-photon'' generation. As shown in the formulas given later in this paper, there are interference terms in the scattering rate. A substitution rule relating electron-wave collisions to the cross-channel process of electron-positron pair production in the collision of an external photon with a two-frequency plane wave includes the effects of ''(multi)photon'' absorption or emission processes in pair production when a photon collides with a two-frequency plane electromagnetic wave. The scattering probabilities of both ''biharmonic'' generation and pair production depend on the relative directions of rotation of the two circularly polarized waves. Relativistic units with $\hbar = c = 1$ are used through the paper.

This paper is organized as follows: In Sec. II, we solve Dirac's equation for an electron in a field of a two-frequency laser beam that is circularly polarized; in Sec. III, we derive probabilities of ''biharmonic-photon'' emission for an electron in such a field. Section IV describes our use of the substitution rule to derive the probabilities of pair production for a photon colliding with a two-frequency plane electromagnetic wave. In Sec. V, we discuss the findings and give our conclusions from the derived formulas.

II. THE WAVE FUNCTION

Since the number of photons in the laser beam is very large, we can treat the field as an external, unquantized electromagnetic field. Suppose the two-frequency electromagnetic wave is circularly polarized, then the field of waves can be described by the four-potential:

$$A=A_1+A_2,$$

$$A_1=a_1\cos\phi_1+a_2\sin\phi_1, \quad \phi_1=(k_1x), \tag{2.1}$$

$$A_2=a_3\cos\phi_2+a_4\sin\phi_2, \quad \phi_2=(k_1x),$$

where $k_{1\mu}$ and $k_{2\mu}$ represent the four-vector propagation (for convenience, we suppose $k_1>k_2$), while $a_{i\mu}$, $i=1$–4, are the amplitudes of the potential. The gauge is chosen such that $(k_1A_1)=(k_2A_2)=0$. We suppose $\vec{k_1}$ is parallel to $\vec{k_2}$, so $\phi_2/\phi_1=\eta=\text{const}<1$ and $k_2=\eta k_1$, the orthogonality conditions are $(a_1a_2)=(a_3a_4)=(k_ia_j)=0$, $i=1,2$, $j=1$–4, while their lengths satisfy $a_1^2=a_2^2, a_3^2=a_4^2$, and $(k_ik_j)=0$, $i,j=1,2$. For the two-frequency plane electromagnetic wave, it is convenient to introduce a dimensionless measure of field strength as is usually done [2–4]:

$$\xi=\frac{e\sqrt{-\langle A_\mu A^\mu\rangle}}{m}, \tag{2.2}$$

where $\sqrt{-\langle A_\mu A^\mu\rangle}$ is the root mean square of the field; then, using Eq. (2.1) we have

$$\xi=\frac{e\sqrt{-a_1{}^2-a_3{}^2}}{m}=\sqrt{\xi_1{}^2+\xi_2{}^2}, \tag{2.3}$$

where ξ_1 and ξ_2 are the field strengths of the two waves, respectively. Recently, nonlinear effects were observed [4] at the laser intensities achieved ($I\approx 10^{18}$ W/cm^2, $\xi\approx 0.6$).

For an electron in a given electromagnetic field, the second-order form of Dirac's equation for the spinor wave function ψ is expressed by

$$\{-\partial^2-2ieA\partial+e^2A^2-m^2-ie \times[(\gamma k_1)(\gamma A_1')+(\gamma k_2)(\gamma A_2')]\}\psi=0, \tag{2.4}$$

where A' is derivative of A. We first suppose $\psi_p=e^{-i(px)}F_1(\phi_1)F_2(\phi_2)$ and put this into Eq. (2.4) following a procedure similar to that described in section 40 of Ref. [12], we find that the exact solution of Dirac's equation has the form

$$\psi_p=\left[1+\frac{e(\gamma k_1)(\gamma A_1)}{2(pk_1)}+\frac{e(\gamma k_2)(\gamma A_2)}{2(pk_2)}\right]\frac{u(p)}{\sqrt{2q_0}} \times\exp\{-i[R_1+R_2+R_3+R_4+(qx)]\}, \tag{2.5}$$

where

$$R_1=\frac{e(a_1p)}{(pk_1)}\sin\phi_1-\frac{e(a_2p)}{(pk_1)}\cos\phi_1,$$

$$R_2=\frac{e(a_3p)}{(pk_2)}\sin\phi_2-\frac{e(a_4p)}{(pk_2)}\cos\phi_2, \tag{2.6}$$

$$R_3=-\frac{e^2}{2(pk_1)}\frac{[b_1^2\sin(\phi_1-\phi_2)-b_2^2\cos(\phi_1-\phi_2)]}{1-\eta},$$

$$R_4=-\frac{e^2}{2(pk_1)}\frac{[b_3^2\sin(\phi_1+\phi_2)-b_4^2\cos(\phi_1+\phi_2)]}{1+\eta}$$

and b_i is defined as

$$b_1{}^2=(a_1a_3)+(a_2a_4), \quad b_2{}^2=(a_2a_3)-(a_1a_4),$$

$$b_3{}^2=(a_1a_3)-(a_2a_4), \quad b_4{}^2=(a_1a_4)+(a_2a_3), \tag{2.7}$$

where $u(p)$ is a bispinor, p_μ is a constant four-vector determining the state, $p^2=m^2$; q is a kind of average four-vector momentum of electron, usually called ''quasimomentum'' of the electron:

$$q_\mu=p_\mu-\frac{a_1^2e^2}{2(pk_1)}k_{1\mu}-\frac{a_3^2e^2}{2(pk_2)}k_{2\mu}, \tag{2.8}$$

so $q^2=m^2(1+\xi^2)=m_*^2$, m_* is the ''effective'' mass of the particle in the field. The factor $1/\sqrt{2q_0}$ in Eq. (2.5) is chosen as the normalization condition that

$$\int \psi_{p'}{}^*\psi_p d^3x=(2\pi)^3\delta(\vec{q'}-\vec{q}). \tag{2.9}$$

From the wave function, we deduce that the electron in the electromagnetic field is no longer free; in fact, we can expand the wave function ψ_p in Eq. (2.5) as (for details see the Appendix)

$$\psi_p=\sum_{s_1s_2s_3s_4} D_{s_1s_2s_3s_4}\frac{u(p)}{\sqrt{2q_0}}\exp\{-i[s_1k_1+s_2k_2+s_3(k_1-k_2) +s_4(k_1+k_2)+q]x\} \tag{2.10}$$

where $D_{s_1s_2s_3s_4}$ is a four-dimensional matrix. All the other general relations investigated in Sec. 2 of Ref. [2] are valid in our case.

III. THE PROBABILITIES OF PHOTON EMISSION BY AN ELECTRON

The S-matrix for emission of an external photon with momentum k' and polarization e' by an electron [see Eq. (73.19) in Ref. [12]] is equal to

$$S_{fi}=-ie\int \bar{\psi}_{p'}(\gamma e'^*)\psi_p\frac{\sqrt{4\pi}e^{ik'x}}{\sqrt{2\omega'}}d^4x, \tag{3.1}$$

where e'^* is a conjugate of e', ω' is the energy of the emitted external photon, and ψ_p and ψ_p' are the initial and final states of the electron, respectively. We use Eq. (2.5), and following the formula (101.7) in Ref. [12], then, simplify Eq. (3.1) as

$$S_{fi}=\frac{1}{\sqrt{2q_0 2q_0' 2\omega'}}\sum_{s_1,s_2,s_3,s_4} M_{fi}{}^{(s_1s_2s_3s_4)}(2\pi)^4\delta^4[q+s_1k_1+s_2k_2+s_3(k_1-k_2)+s_4(k_1+k_2)-q'-k'], \tag{3.2}$$

where s_1, s_2, s_3, and s_4 are integers. For an electron in monochromatic intense plane electromagnetic wave, Nikishov and Ritus explained Eq. (11) in Ref. [2] as the absorption or emission of integer number of photons, if we follow their argument we may extend the annotation to our case, in which we deduce that those s_i are the number of photons absorbed from the wave or emitted to the wave by the electron; however, because we use the classical treatment of initial electromagnetic wave, using the word photon may not be clear, so the above argument may be modified (corrected) as follows: s_1, s_2, s_3, and s_4 are the integer multiples of momenta k_1, k_2, k_1-k_2, and k_1+k_2, respectively, which are absorbed or emitted by the electron. This signifies that the electron can only absorb or emit discrete momenta from the electromagnetic wave. For this reason, and sometimes for convenience, we still use ''photon'' or ''multiphoton'' but in quotes, instead of using the discrete momentum of the electromagnetic wave. In this sense, we conclude that ''multiphoton'' processes may occur; we discuss this in more detail later. From the representation of the δ function in Eq. (3.2), we obtain the following kinematic equation:

$$q+s_1k_1+s_2k_2+s_3(k_1-k_2)+s_4(k_1+k_2)=q'+k'. \tag{3.3}$$

In the frame in which the electron is at rest, on average, ($\vec{q}=0$, $q_0=m_*$), $(qk_1)=m_*\omega_1$, $(qk')=m_*\omega'$, $(k_1k')=\omega_1\omega'(1-\cos\theta)$, and θ is the scattering angle between $\vec{k}_1$ and $\vec{k}'$. With these expressions and the kinematic equation (3.3), we can assess emitted photon frequency:

$$\omega'=\frac{s_1\omega_1+s_2\omega_2+s_3(\omega_1-\omega_2)+s_4(\omega_1+\omega_2)}{1+(v\omega_1/m_*)(1-\cos\theta)}, \tag{3.4}$$

where ω_1 and ω_2 are the frequencies of two incident electromagnetic waves in this frame, and

$$v=s_1+\eta s_2+(1-\eta)s_3+(1+\eta)s_4. \tag{3.5}$$

When the energy of the incident electron is low ($\gamma\approx1$) and frequency of the wave also is low ($\omega_1\sim1$ eV in the laboratory frame), then Doppler effects are negligible, so we have $\omega_1/m\ll1$, and Eq. (3.4) can be simplified as

$$\omega'=n_1\omega_1+n_2\omega_2, \tag{3.6}$$

where $n_1=s_1+s_3+s_4$ and $n_2=s_2-s_3+s_4$; this is the nonrelativistic limit of the biharmonic-photon. For an incident electron with high energy ($\gamma\gg1$), ω_1 is Doppler shifted and ω_1/m_* may be not small, but even if this term is negligible the final-state photon k' is Doppler shifted from its biharmonic value; so, in this relativistic limit, the emitted photon is no longer (bi)harmonic. This is the reason why we enclose the terms ''(bi)harmonics'' or ''(bi)harmonic-photon'' in quotation marks.

If we sum the polarizations of the inital and final electrons, and of the emitted external photons for the square of the matrix element $M_{fi}^{(s_1s_2s_3s_4)}$ in Eq. (3.2), and use the expansion formula from Eq. (101.7) in Ref. [12], then, after a very complicated derivation we find

$$\begin{aligned}\sum_{\text{polar}}|M_{fi}^{(s_1s_2s_3s_4)}|^2=&4\pi e^2m^2J_{s_3}{}^2(z_3)J_{s_4}{}^2(z_4)J_{s_1}{}^2(z_1)J_{s_2}{}^2(z_2)\left\{-4+\left[2+\frac{(k_1k')^2}{(pk_1)(p'k_1)}\right]\right.\\&\times\left[4\frac{(1-\eta)s_3+(1+\eta)s_4}{m^2}\frac{(pk_1)(p'k_1)}{(k_1k')}-\xi_1{}^2\frac{2J_{s_1}{}^2(z_1)-J_{s_1+1}{}^2(z_1)-J_{s_1-1}{}^2(z_1)}{J_{s_1}{}^2(z_1)}\right.\\&-\xi_2{}^2\frac{2J_{s_2}{}^2(z_2)-J_{s_2+1}{}^2(z_2)-J_{s_2-1}{}^2(z_2)}{J_{s_2}{}^2(z_2)}-\frac{e^2}{m^2}\frac{J_{s_1+1}(z_1)J_{s_2+1}(z_2)+J_{s_1-1}(z_1)J_{s_2-1}(z_2)}{J_{s_1}(z_1)J_{s_2}(z_2)}\\&\left.\left.\times(b_1^2\cos\phi_{12}+b_2^2\sin\phi_{12})-\frac{e^2}{m^2}\frac{J_{s_1+1}(z_1)J_{s_2-1}(z_2)+J_{s_1-1}(z_1)J_{s_2+1}(z_2)}{J_{s_1}(z_1)J_{s_2}(z_2)}(b_3^2\cos\Phi+b_4^2\sin\Phi)\right]\right\},\end{aligned} \tag{3.7}$$

where $J_s(z)$ is the Bessel function of order s, and

$$\phi_{12}=\phi_{10}-\phi_{20},\quad \Phi=\phi_{10}+\phi_{20},\quad \tan\phi_{10}=\frac{\alpha_2}{\alpha_1},\quad \tan\phi_{20}=\frac{\alpha_4}{\alpha_3},$$

$$z_1=\sqrt{\alpha_1^2+\alpha_2^2},\quad z_2=\frac{\sqrt{\alpha_3^2+\alpha_4^2}}{\eta},\quad z_3=\frac{\sqrt{\beta_1^2+\beta_2^2}}{1-\eta},\quad z_4=\frac{\sqrt{\beta_3^2+\beta_4^2}}{1+\eta},$$

$$\alpha_i=e\left[\frac{(a_ip)}{(pk_1)}-\frac{(a_ip')}{(p'k_1)}\right],\quad \beta_i=\frac{e^2}{2}\left[\frac{1}{(pk_1)}-\frac{1}{(p'k_1)}\right]b_i{}^2,\quad i=1-4. \tag{3.8}$$

The probability of ‘‘biharmonic-photon’’ generation by an electron per unit time is

$$w^{\{s_1,s_2,s_3,s_4\}}=\int |M_{fi}^{(s_1s_2s_3s_4)}|^2\frac{d^3k'd^3q'}{(2\pi)^6 2q_0 2q_0{}' 2\omega'}(2\pi)^4\delta^4[q+s_1k_1+s_2k_2+s_3(k_1-k_2)+s_4(k_1+k_2)-q'-k']. \tag{3.9}$$

Then, the total scattering rate is

$$W=\sum_{s_1s_2s_3s_4} w^{\{s_1,s_2,s_3,s_4\}}. \tag{3.10}$$

Suppose the energy emission rate of the ‘‘biharmonic-photon’’ is $I^{\{s_1,s_2,s_3,s_4\}}$, then it can be simply obtained by multiplying ω' by the integrand of Eq. (3.9),

$$I^{\{s_1,s_2,s_3,s_4\}}=\int |M_{fi}{}^{(s_1s_2s_3s_4)}|^2\frac{d^3k'd^3q'}{(2\pi)^6 8q_0q_0{}'}(2\pi)^4\delta^4[q+s_1k_1+s_2k_2+s_3(k_1-k_2)+s_4(k_1+k_2)-q'-k']. \tag{3.11}$$

The expression in Eq. (3.7) can be simplified by using the polarization property of the electromagnetic wave. Since the two-frequency plane electromagnetic wave is circularly polarized, these two waves have either like or opposite helicities. Thus, there are two cases; first, that of two circularly polarized electromagnetic waves, which have the same direction of rotation, that is, they are both left-circular-polarized or right-circular-polarized; the other case is where these two waves have opposite directions of rotation. In both cases, the field of waves can be expressed as

$$\begin{gathered} A=A_1+A_2,\\ A_1=a_1\cos\phi_1+a_2\sin\phi_1,\quad \phi_1=(k_1x),\\ A_2=\zeta[a_1\cos(\phi_2+\varphi)\pm a_2\sin(\phi_2+\varphi)],\quad \phi_2=(k_2x), \end{gathered} \tag{3.12}$$

where φ is a phase difference between these two plane waves; the plus in $\pm$ in the above equation corresponds to like helicities, and the minus corresponds to opposite helicities. The factor ζ defines the ratio of field strength between the two laser beams, so if the field strength of the wave k_1 is ξ_1, then the field strength of the other wave is $\xi_2=\zeta\xi_1$, and the total strength of field is $\xi=\xi_1\sqrt{(1+\zeta^2)}$ from Eq. (2.3). For two circularly polarized waves with like helicities, Eq. (2.7) gives $b_3=b_4=0$ for any phase φ; from Eqs. (2.5) and (2.6) we can see the component of k_1+k_2 in the wave function has disappeared, so the kinematic equation is reduced to

$$q+s_1k_1+s_2k_2+s_3(k_1-k_2)=q'+k'. \tag{3.13}$$

For opposite helicities, $b_1=b_2=0$ and so there is no k_1-k_2 component remaining in the wave function and kinematic equation is

$$q+s_1k_1+s_2k_2+s_4(k_1+k_2)=q'+k'. \tag{3.14}$$

We put these two conditions together, then:

$$q+s_1k_1+s_2k_2+s_l(k_1\mp k_2)=q'+k', \tag{3.15}$$

where $s_l=s_3$ for like helicities and $s_l=s_4$ for opposite helicities, and the upper sign in $\mp$ or $\pm$ always corresponds to like helicities, while the lower sign corresponds to opposite helicities. We use these definitions through the paper. If we take the invariant $u=(k_1k')/(k_1p')$, then from Eq. (3.9), the probabilities for above two cases reduce to

$$\begin{aligned} w_\pm{}^{\{s_1,s_2,s_l\}}=\frac{e^2m^2}{4q_0}\int_0^{u_v}\frac{du}{(1+u)^2}J_{s_1}{}^2(z_1)J_{s_2}{}^2(z_2)J_{s_l}{}^2(z_l)\Bigg\{-4+\xi_1{}^2\left[2+\frac{u^2}{(1+u)}\right]\Bigg[-2(1+\zeta^2)+2\zeta\frac{J_{s_l+1}(z_l)+J_{s_l-1}(z_l)}{J_{s_l}(z_l)}\\ +\left(\frac{J_{s_1+1}(z_1)}{J_{s_1}(z_1)}+\zeta\frac{J_{s_2\pm1}(z_2)}{J_{s_2}(z_2)}\right)^2+\left(\frac{J_{s_1-1}(z_1)}{J_{s_1}(z_1)}+\zeta\frac{J_{s_2\mp1}(z_2)}{J_{s_2}(z_2)}\right)^2\Bigg]\Bigg\}, \end{aligned} \tag{3.16}$$

where

$$u_v = \frac{2v(k_1 p)}{m_*{}^2}, \quad v = s_1 + \eta s_2 + (1 \mp \eta) s_l,$$

$$z_1 = 2v \frac{\xi_1}{\sqrt{1+\xi^2}} \sqrt{\frac{u}{u_v}\left(1 - \frac{u}{u_v}\right)}, \quad z_2 = \frac{\zeta z_1}{\eta}, \quad z_l = \frac{2v}{1 \mp \eta} \frac{\zeta \xi_1{}^2}{1+\xi^2} \frac{u}{u_v}, \tag{3.17}$$

and z_1, z_2, and z_l are calculated using the formulas in Sec. 101. of Ref. [12]. The results for both cases are independent of the phase φ in Eq. (3.12); an interesting point is that the two waves can interact simultaneously with an electron, no matter what the phase difference is between them.

Formula (3.16) should include the case of a monochromatic wave; this can easily be checked by putting $\zeta = 0$ into this equation (3.16). We obtain exactly the same formula as that in Ref. [3]. If we expand formulas (3.16) in powers of ξ_1 and ζ when $\xi_1 \ll 1$ and $\xi_2 \ll 1$, we can obtain the limit of a weak electromagnetic field. As expected, the term of order $\xi_1{}^2$ is a simple sum of the Klein-Nishina formula (see section 86 in Ref. [12]) of the individual waves. The higher term of order ξ_1^4 involves many parts; here, we give the formula of the interference part, that is, $s_1 = 0$, $s_2 = 0$ and $s_l = 1$ (for like helicities $s_l = s_3 = 1$ and $s_4 = 0$, for opposite helicities $s_l = s_4 = 1$ and $s_3 = 0$) part in Eq. (3.16):

$$W_{\text{inter}} = \frac{e^2 m^2}{q_0} \xi_1{}^4 \zeta^2 \left\{ \left(\frac{1}{2} - \frac{2}{u_{12}} \right) [1 - \ln(1 + u_{12})] + \frac{5}{4(1+u_{12})} - \frac{1}{4(1+u_{12})^2} \right\}, \tag{3.18}$$

where

$$u_{12} = \frac{2[(k_1 p) - (k_2 p)]}{m_*{}^2}. \tag{3.19}$$

Generally, the ξ^4 term is a nonlinear contribution, so inteference effects are nonlinear effects.

IV. THE PROBABILITIES OF PAIR PRODUCTION BY A PHOTON

If we substitute $p \to -p$, $k' \to -k_\gamma$ and $d^3k' \to d^3q$ in Eqs. (3.9) and (3.16), and reverse the common sign of the expression, we can obtain the probability per unit time of pair production by a photon of momentum k_γ in a two-frequency plane electromagnetic wave:

$$w_\pm{}^{\{s_1,s_2,s_l\}} = \frac{e^2 m^2}{4k_{\gamma 0}} \int_1^{u_v} \frac{du}{u\sqrt{u(u-1)}} J_{s_1}^2(z_1) J_{s_2}^2(z_2) J_{s_l}^2(z_l) \left\{ 2 + \xi^2(2u-1) \left[-2(1+\zeta^2) + 2\zeta \frac{J_{s_l+1}(z_l) + J_{s_l-1}(z_l)}{J_{s_l}(z_l)} + \left(\frac{J_{s_1+1}(z_1)}{J_{s_1}(z_1)} + \zeta \frac{J_{s_2 \pm 1}(z_2)}{J_{s_2}(z_2)} \right)^2 + \left(\frac{J_{s_1-1}(z_1)}{J_{s_1}(z_1)} + \zeta \frac{J_{s_2 \mp 1}(z_2)}{J_{s_2}(z_2)} \right)^2 \right] \right\} \tag{4.1}$$

and

$$u = \frac{(k_1 k_\gamma)^2}{4(k_1 q)(k_1 q')}, \quad u_v = \frac{v(k_1 k_\gamma)}{2m_*^2}, \quad v > v_s = \frac{2m_*^2}{(k_1 k_\gamma)}. \tag{4.2}$$

z_1, z_2, z_l, and v have the same expression as in Eq. (3.17). In this case, the kinematic equation is

$$k_\gamma + s_1 k_1 + s_2 k_2 + s_l(k_1 \mp k_2) = q + q', \tag{4.3}$$

where q and q' are the "quasimomentum" of the electron and positron, respectively. The s_l has the same meaning as in Eq. (3.15) and is connected with interference effects. When we put $\zeta = 0$ into the above formula, it reduces to the case of a monochromatic wave.

In the center-of-mass system (in which $\vec{k}_\gamma + v\vec{k}_1 = \vec{q} + \vec{q}\,' = 0$), the energy of electron and positron can be easily obtained:

$$q_0 = q'_0 = \sqrt{\frac{v}{2}(k_\gamma k_1)}. \tag{4.4}$$

For the case of $\xi \ll 1$, it corresponds to the weak field limit and we can expand the formula (4.1) in powers of ξ_1 and ζ when $\xi_1 \ll 1$ and $\xi_2 \ll 1$. The term of order ξ_1^2 is the simple sum of the Breit-Wheeler formula [13] of individual waves, as expected; the term ξ_1^4 gives a nonlinear contribution; here, we present only the interference effect in this term, which is the case of $s_1 = 0$, $s_2 = 0$, $s_l = 1$,

$$W_{\rm inter}=\frac{e^2m^2}{4k_{\gamma 0}}\frac{\xi_1{}^4\zeta^2}{u_{12}{}^2}\{2(1+2u_{12})\sqrt{u_{12}(u_{12}-1)}+\ln[2u_{12}-1+2\sqrt{u_{12}(u_{12}-1)}]\}, \tag{4.5}$$

where it needs

$$u_{12}=\frac{(k_1k_\gamma)\mp(k_2k_\gamma)}{2m_*{}^2}>1. \tag{4.6}$$

From condition (4.2) we deduce that a higher-energy photon will more easily produce electron pairs because as the photon's energy increases v_s becomes smaller; then, lower-order pair production processes occur (corresponding to a small v). On the other hand, for any given set of integers s_i, there is a threshold of pair production, but if the integers s_i are allowed to be arbitrarily large, the threshold can be arbitrarily low. This is nonlinear effect and it is always negligible for the case of weak wave [14].

Another interesting phenomenon of the interference effect is that in the case of opposite helicities it is easier to assign an interference contribution to pair production than in the case of like helicities. For example, an incident photon with energy $\omega_\gamma\sim 100$ GeV propagates in an opposite direction to an intense two-frequency laser beam with one frequency $\omega_1\sim 2$ eV and $\eta\sim 0.5$, with dimensionless field strength $\xi\sim 0.5$, which corresponds to the strength of the magnetic field in the center-of-mass system $B_{\rm max}\sim\xi B_0$, where B_0 is the critical strength of the magnetic field, $B_0=m^2c^3/e\hbar$; in this case, $v_s\sim 1.9$, if $s_1=1$, $s_2=0$, $s_l=1$, then for the case of like helicities $v=s_1+s_2\eta+s_l(1-\eta)=1.5<v_s$ and it has no contribution to pair production; however, for opposite helicities $v=s_1+s_2\eta+s_l(1+\eta)=2.5>v_s$ and its probabilities can be evaluated from Eq. (4.1), which involves interference effect. This phenomenon that the lower rate for pair production near threshold for like helicities may be explained by angular momentum barrier effect, because the photons have like helicities their total spin is larger.

V. DISCUSSION AND CONCLUSIONS

In Secs. III and IV, we formulated the probabilities of ''biharmonic-photon'' generation and of pair production. The total probabilities of scattering are the simple sums of Eqs. (3.16) or (4.1),

$$W_\pm=\sum_{s_1,s_2,s_l}w_\pm{}^{\{s_1,s_2,s_l\}}. \tag{5.1}$$

For the case of an electron in the two-frequency plane electromagnetic wave, each term in this equation corresponds to the probability of the emission of a photon with frequency ω' in the rest frame that is defined in Eq. (3.15). For the case of the pair production, each term in this equation corresponds to the the probability of generating the particles of the pair with the energy given in Eq. (4.4) in the center-of-mass system. Therefore, the different combinations of $\{s_1,s_2,s_l\}$ correspond to different modes, and every mode has its relevant probability.

In Sec. III, we gave the meaning of s_i, $i=1$–4; here, we discuss it further. For convenience, we assume $k_3=k_1-k_2$, $k_4=k_1+k_2$, then $s_i>0$ (or $s_i<0$) corresponds to the occurences of processes of absorption (or emission) of $|s_i|$ times the momentum k_i from waves by the electron, that is, only the discrete four-vector momentum of the electromagnetic wave can be absorbed or emitted by the electron. In this sense, we may say that the electron absorbs (or emits) ''photon(s).'' When $s_i<0$, the ''photon'' $(s_i=-1)$ or ''multiphoton'' $(s_i<-1)$ is emitted by the electron into the wave at exactly the wave frequency. The emission effects here are a novel kind of stimulated emission in which ''photon(s)'' are emitted by an electron into one laser beam due to the presence of another of different frequency. This contrasts with the more usual configuration involving a single laser frequency [15]. The two-frequency electromagnetic wave is treated in a classical way, in this analysis it does not contain photons. After solving the Dirac equation, we find an electron in a plane electromagnetic wave can absorb or emit only discrete 4-momenta from and to the two-frequency wave, that is to say, the processes of the absorption (or emission) of an electromagnetic wave by the electron conform to a quantum mode, and that the wave absorbed or emitted with discrete 4-momentum is a wave quantum(s), that is, a photon(s).

In the interference processes, the absorption or emission of the wave with momentum $k_1\mp k_2$ is just like k_1 or k_2; therefore, we can say that the electron absorbs or emits $|s_l|$ ($s_l=s_3$, $s_4=0$, if like helicities, and $s_l=s_4$, $s_3=0$, if opposite helicities) number of ''photons'' of momentum $k_1\mp k_2$. Though at first there are no waves with momentum $k_1\mp k_2$, from Eq. (3.3) it seems there is no difference between momentum $k_1\mp k_2$ and individual momentum k_1, k_2. If we suppose the ''photons'' of momentum $k_1\mp k_2$ are really like individual photons, then this interpretation leads to interesting phenomena, such that when $s_l<0$ there should be $|s_l|$ number of ''photons'' $k_1\mp k_2$ emitted (these ''photons'' are not final-state photons k'), which are not Doppler shifted. For the case of an intense wave $(\xi\sim 1)$, we may expect that those emitted waves with mometum $k_1\mp k_2$ are observable, that is, when a two-frequency electromagnetic wave has like helicities, we may find a third color with frequency $\omega_1-\omega_2$ in the wave; when the helicities are opposite a third color with frequency $\omega_1+\omega_2$ is expected. On the other hand, from Eq. (3.18), the contribution from the $k_1\mp k_2$ part is of the order of ξ^4 or higher, in this sense, the momentum $k_1\mp k_2$ may have no special meaning, it just corresponds to interference corrections, so the phenomena just mentioned above may be impossible. We cannot be sure which interpretation is correct, the full-quantum treatment of an electron in an intense plane wave may clarify matters. The above arguments are also true for the case of pair production.

For both ''biharmonic'' generation and pair production, we classify physical events by mode $\{s_1,s_2,s_l\}$, not by final state. Sometimes, the different mode corresponds to the same final state, but they are different. For example, in the case that $k_1=2k_2$, the mode with $s_1=2$, $s_2=1$, $s_3=0$, and $s_4=-1$ (opposite helicities), and the mode with $s_1=1$, $s_2=s_3=s_4=0$ have the same $v=1$, which corresponds to the same energy of final-state photon ω' in Eq. (3.4), or similarly, for final-state particles of pair p_0 and p'_0 in Eq. (4.4), but they have, at least, one difference: they correspond to different probabilities that can be calculated with Eqs. (3.16) or (4.1).

The effects discussed above are based on results derived from Dirac's equation where the field of a laser beam is described in an unquantized classical way. When a two-frequency laser beam is scattered by electrons, there are various ''biharmonic-photons'' emission processes that follow ''photon'' or ''multiphoton'' absorption and stimulated ''(multi)photon'' emission processes; the scattering probabilities are affected by the effects of interference between two laser beams. Similar effects are involved in pair production by the interaction of an external γ photon with a two-frequency laser beam.

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APPENDIX

We can express wave function (2.5) as

$$\psi_p=[1+f_1(\phi_1)+f_2(\phi_2)] \times\exp\{-i[R_1+R_2+R_3+R_4+(qx)]\}\frac{u(p)}{\sqrt{2q_0}}, \tag{A1}$$

where $f_1(\phi_1)=e(\gamma k_1)(\gamma A_1)/2(pk_1)$ and $f_2(\phi_2)=e(\gamma k_2)(\gamma A_2)/2(pk_2)$, they are 4×4 matrix, then we use the expansion

$$f_1(\phi_1)\exp(-iR_1)=\sum_{s_1} B_{s_1}\exp(-is_1\phi_1),$$

$$f_2(\phi_2)\exp(-iR_2)=\sum_{s_2} B_{s_2}\exp(-is_2\phi_2), \tag{A2}$$

where coefficients B_{s_1} and B_{s_2} are 4×4 matrixes; they can be calculated by

$$B_{s_1}=\frac{1}{2\pi}\int_0^{2\pi}d\phi_1 f_1(\phi_1)\exp[-i(R_1-s_1\phi_1)],$$

$$B_{s_2}=\frac{1}{2\pi}\int_0^{2\pi}d\phi_2 f_2(\phi_2)\exp[-i(R_2-s_2\phi_2)], \tag{A3}$$

and

$$\exp(-iR_i)=\sum_{s_i} C_{s_i}\exp(-is_i\phi_i), \quad i=1-4,$$

$$C_{s_i}=\frac{1}{2\pi}\int_0^{2\pi}d\phi_i\exp[-i(R_i-s_i\phi_i)], \tag{A4}$$

where $\phi_3=\phi_1-\phi_2$ and $\phi_4=\phi_1+\phi_2$, then, the wave function becomes

$$\psi_p=\sum_{s_1s_2s_3s_4} D_{s_1s_2s_3s_4}\frac{u(p)}{\sqrt{2q_0}}\exp\{-i[s_1\phi_1+s_2\phi_2 +s_3(\phi_1-\phi_2)+s_4(\phi_1+\phi_2)+(qx)]\}, \tag{A5}$$

where $\phi_1=(k_1x)$ and $\phi_2=(k_2x)$, and matrix $D_{s_1s_2s_3s_4}$ is

$$D_{s_1s_2s_3s_4}=C_{s_3}C_{s_4}(C_{s_1}C_{s_2}+C_{s_1}B_{s_2}+C_{s_2}B_{s_1}). \tag{A6}$$

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